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Ensemble-based Data Assimilation For High-uncertainty systems: Case of study, PM10 and PM2.5 in the Aburrá Valley

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## Outline

- Introduction
- LOTOS-EUROS Model
- Data Assimilation
- Preliminary Results

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#### Background

Time: 2016-04-01 00:00:00



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Vigilada Mineducación



**EnKF** 





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Date



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#### Just Model Emissions PM2.5 Data Assimiliation Emissions PM2.5



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#### Just Model Emissions Means



#### **Data Assimilation Emissions Means**



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# **Research Description**

The KF is an optimal method when different assumptions about the statistical behavior of the uncertainties are met.

But in many real applications there are not enough information to characterize the system uncertainties or there are too many uncertainties sources.

It is proposed a three-year plan to develop a Ensemble-based data assimilation scheme to lead with high-uncertainty and high-dimensional systems.

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# **Research Description**

The scheme will be focus in three important aspects of the data assimilation process.



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## **Research Description**



Schematic representation of parameter estimation using a model to propagate the emission uncertainty and a Ensemble-based DA scheme. Based in (Peng et al., 2017).

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- In the EnKF it is necessary to make some statistical assumptions related to the uncertainty in the model and the observations, that in many of real applications are no truth. For instance, the Gaussian distribution of the state error.
- A different approach when the systems condition does not satisfy the requirement of the KF-based methods are the robust filters or robust estimators.
- The robust filters emphasize the robustness of the estimation, so that they may have better tolerances to possible uncertainties in assimilation. Since its purpose is not the optimality in the estimation, the robust estimator does not require an exactly statistical representation of the error, showing a better performance that the KF-based methods in scenarios with poor statistical representation of the uncertainty.

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• Unlike the KF that minimize the variance of the estimation error, the HF is based on the criterion of minimizing the supremum of the  $L_2$  norm of the uncertainty sources (initial conditions, parameters, boundary conditions, etc.) (Han et al., 2009). The HF requires that the total energy of the estimation errors, be no longer than the uncertainty source energy times a factor  $1/\gamma$ :

$$\sum_{t=0}^{K} \left| \left| x_{t}^{t} - x_{t}^{a} \right| \right|_{S_{t}}^{2} \leq \frac{1}{\gamma} \left( \left| \left| x_{0}^{t} - x_{0}^{a} \right| \right|_{\Delta_{0}^{-1}}^{2} + \sum_{t=0}^{K} \left| \left| u_{t} \right| \right|_{Q_{t}^{-1}}^{2} + \sum_{t=0}^{K} \left| \left| v_{t} \right| \right|_{R_{t}^{-1}}^{2} \right)$$
(1)

Where  $x_t^t$  is the truth state,  $x_t^a$  is the analysis state,  $S_t$  is a user-chosen matrix of weights,  $u_t$  and  $v_t$  are the model and observation uncertainty,  $\Delta_t$ ,  $Q_t$  and  $R_t$  are the uncertainty weights matrices with respect to the initial conditions, model error and observations error.

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To solve equation (1), it is defined first the following cost function  $J^{HF}$ :

$$J^{HF} = \frac{\sum_{t=0}^{K} \left| \left| x_{t}^{t} - x_{t}^{a} \right| \right|_{S_{t}}^{2}}{\left| \left| x_{0}^{t} - x_{0} \right| \right|_{\Delta_{0}^{-1}}^{2} + \sum_{t=0}^{K} \left| \left| u_{t} \right| \right|_{Q_{t}^{-1}}^{2} + \sum_{t=0}^{K} \left| \left| v_{t} \right| \right|_{R_{t}^{-1}}^{2}}$$

Then the inequality (1) is equivalent to  $J^{HF} \leq \frac{1}{\gamma} \operatorname{Let} \gamma^*$  be the value such that:

$$\frac{1}{\gamma^*} = \frac{\inf \sup_{\boldsymbol{x_t^a x_0, \{u_t\}, \{v_t\}}} J^{HF}, t \le K$$

The optimal HF is achieved when  $\gamma = \gamma^*$ . In this sense, the evaluation of  $\gamma *$  is an application of the minimax rule, a strategy that aims to provide robust estimates and is different from its Bayesian counterpart.

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The  $\gamma$  is then know as performance level of the HF. The inequality (1) can be solve iteratively, similar that in the KF:

$$\begin{aligned} x_{t}^{f} &= M_{t-1}(x_{t-1}^{a}) \\ \Delta_{t}^{f} &= M_{t-1}\Delta_{t-1}^{a}M_{t-1}^{T} + Q_{t} \\ (\Delta_{t}^{a})^{-1} &= \left(\Delta_{t}^{f}\right)^{-1} + H^{T}(R_{t})^{-1H} - \gamma S_{t} \\ G_{t} &= \Delta_{t}^{aH^{T}}(R_{t})^{-1} \\ x_{t}^{a} &= x_{t}^{f} + G_{t}(y_{t} - Hx_{t}^{f}) \end{aligned}$$

Subscript to the constraint:

$$(\Delta_t^a)^{-1} = (\Delta_t^f)^{-1} + H^T(R_t)^{-1}H - \gamma S_t \ge 0.$$

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where  $\Delta$  denotes the uncertainty matrix, analogues to the covariance matrix *P* in the KF, and *G*<sub>t</sub> is the HF Gain matrix analogues to the Kalman Gain *K*<sub>t</sub>. To compare directly the HF and the KF, let rewrite:

$$P_t^a = (I - K_{tH_t}) P_t^f.$$

as:

$$(P_t^a)^{-1} = \left(P_t^f\right)^{-1} + H^T(R_t)^{-1}H$$

In addition, let

$$(\Sigma_t^a)^{-1} = (\Delta_t^f)^{-1} + H^T(R_t)^{-1}H$$

Then, it is clear that  $\Sigma_t^a$  is a uncertainty matrix created updating  $\Delta_t^f$  trough a KF, where:

$$(\Delta_t^a)^{-1} = (\Sigma_t^a)^{-1} - \gamma S_t < (\Sigma_t^a)^{-1}$$

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The EnLTHF proposed in (Luo and Hoteit, 2011) is a time-local version of HF which utilizes only the current state and observations of the system rather than the entire available history (Nan and Wu, 2017). Unlike the HF where the cost function  $J^{HF}$  is defined in all the assimilation windows, in the EnLTHF a local cost function is proposed:

$$J_t^{HF} = \frac{\left| |x_t^t - x_t^a| \right|_{S_t}^2}{\left| |x_0^t - x_0| \right|_{\Delta_0^{-1}}^2 + \left| |u_t| \right|_{Q_t^{-1}}^2 + \left| |v_t| \right|_{R_t^{-1}}^2}$$

Similarly to equation (1), it is required that:

$$\left| \left| x_t^t - x_t^a \right| \right|_{S_t}^2 \le \frac{1}{\gamma_t} \left( \left| \left| x_0^t - x_0^a \right| \right|_{\Delta_0^{-1}}^2 + \left| \left| u_t \right| \right|_{Q_t^{-1}}^2 + \left| \left| v_t \right| \right|_{R_t^{-1}}^2 \right)$$

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where  $\gamma_t$  is a suitable local performance level, which satisfies:

$$\frac{1}{\gamma_t} \ge \frac{1}{\gamma_t^*} = \frac{\inf f \quad \sup}{\boldsymbol{x_t^a} \quad x_0, \{\boldsymbol{u}_t\}, \{\boldsymbol{v}_t\}} \quad J_t^{HF}, t \le K$$

with  $\frac{1}{\gamma_t^*}$  being the minimax point of the local cost function  $J_t^{HF}$ .

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The EnLTHF can be expressed in terms of the EnKF algorithm using the notation of (Luo and Hoteit, 2011)

$$\begin{split} [\Sigma_t^a, K_t] &= EnKF(x_t^a, Q_t, H) \\ G_t &= [I_m - \gamma_t \Sigma_t^a S_t]^{-1} K_t \\ \xi_t^{a(i)} &= \xi_t^{f(i)} + G_t \left[ y_t - H_t \xi_t^{f(i)} + v_t^i \right] \\ x_t^a &= \left( \sum_{i=1}^N \xi_t^{a(i)} \right) / N \\ \Delta_t^a &= [I_m - \gamma_t \Sigma_t^a S_t]^{-1} K_t \end{split}$$

subject to the constraint

$$(\Delta_t^a)^{-1} = (\Sigma_t^a)^{-1} - \gamma_t S_t \ge 0$$

where the operator EnKF() means that  $\Sigma_t^a$  and  $K_t$  are obtained through the EnKF.

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#### MONTHLY WEATHER REVIEW

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#### Robust Ensemble Filtering and Its Relation to Covariance Inflation in the Ensemble Kalman Filter

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(Manuscript received 1 December 2010, in final form 11 April 2011)

#### Numerical experiment with the model Lorenz 96

$$\frac{dx_i}{dt} = (x_{i+1} - x_{i-2})x_{i-1} - x_i + F(t), \ i = 1, \dots, 40$$

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- The idea of distance-dependent localization technique where the localization radius can vary according with knowledge about the system has been little studied in atmospheric DA applications.
- In History Matching problems, there are more applications that try to do something related with theses ideas. Specifically in Soares et al., 2018) two different localization methods in a reservoir parameter estimation case are compared. In the first one, are used influences areas of the observations and delimit the localization windows. The second method is based in streamlines and selection of the historical time when they presented the biggest area and trace the influence area
- Localization technique based in streamlines simulation are proposed as an alternative to the distance-dependent methods and using more efficiently the model information.

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**Distance-dependent localization** 

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Streamlines simulation localization

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- In most application of parameter estimation and uncertainty modelling using Ensemble-based DA are followed two approaches: to model the uncertainty as a stochastic process (colored noise) like is described in (Heemink and Segers, 2002) or as a combination of old value like in (Peng et al., 2017).
- The two methods are explain next taking as example the emission estimation in CTMs applications.

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In (Heemink and Segers, 2002) the deterministic model state is represented in discrete time as:

$$x_t = M(x_{t-1})$$

Since the emissions are an important source or error, the uncertainty in the emission are modeled as a stochastic process, for this case, as a colored noise (Jazwinski, 1970):

$$\delta e_t = \alpha \delta e_{t-1} + \sqrt{1 - \alpha^2} w_t$$

where  $w_t$  is a white noise and  $\delta e_t$  is the emission correction factor. Thus, he stochastic model state is formed by augmenting the state vector with the correction factor  $\delta e_t$ :

$$\begin{bmatrix} x_t \\ \delta e_t \end{bmatrix} = \begin{bmatrix} M(x_{t-1}) \\ \alpha \delta e_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \sqrt{1 - \alpha^2} \end{bmatrix} w_t$$

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On the other hand, the method proposed in (Peng et al., 2017) uses a persistence forecasting operator that serve as the forecast model for the emission correction factors. This forecast model is built by a smooth operator using the state ensemble and the previous analysis value of emission correction factors  $\lambda_t^a$ :

$$\lambda_t^{p(i)} = \frac{\xi_t^{f(i)}}{x_t^f}$$
$$\lambda_t^{f(i)} = \frac{1}{T} \left( \sum_{j=t-T+1}^{t-1} \lambda_j^{a(i)} + \lambda_t^{p(i)} \right)$$

Here, T is the time windows of the smooth operator.

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The state vector is augmented with the correction factors  $\lambda$  and can be estimated through Ensemble-based DA. With this, is created a forecast emission ensemble and an analysis forecast ensemble following:

$$\hat{e}_t^f(i) = \lambda_t^{f(i)} e_t$$
$$\hat{e}_t^a(i) = \lambda_t^{a(i)} e_t$$

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