

# Inspire Create Transform

# The Variational approach on the road to Data Assimilation (DA) for Chemical Transport Models (CTM).



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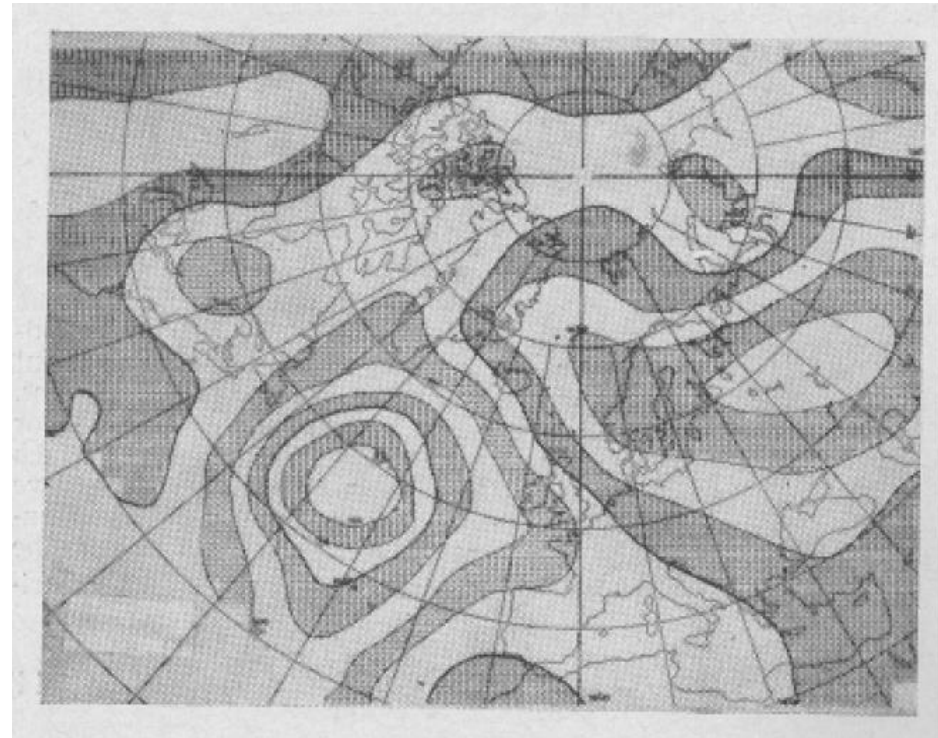
Medellín, Colombia  
2017



# Outline

- Motivation
- Introduction
- Data assimilation fundamentals
- Variational approach
- Previous results
  - TU Delft theses
- Current research questions and future work
- References

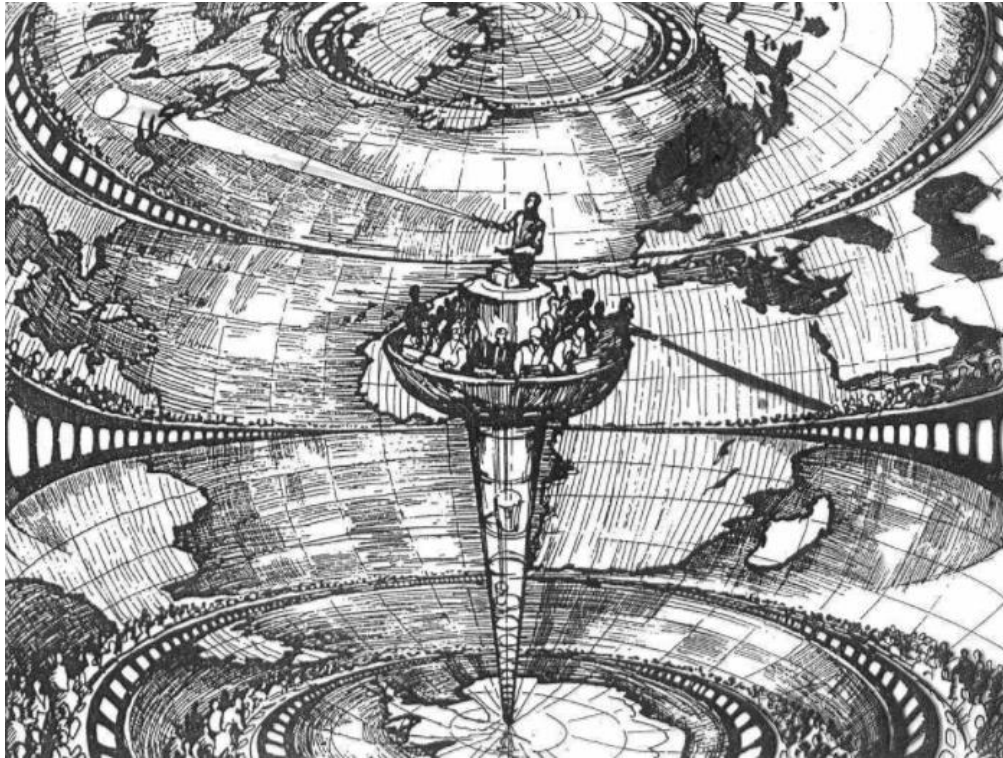
# Motivation



For Bergeron, weather map analysis was rather a fine art than applied science.

(Persson, 2004)

# Motivation



This illustration depicts Richardson's "forecast factory." *Image courtesy of L. Bengtsson.*

[https://celebrating200years.noaa.gov/foundations/numerical\\_wx\\_pred/theater.html](https://celebrating200years.noaa.gov/foundations/numerical_wx_pred/theater.html)

1922

# Motivation



*The Cray supercomputer used for weather analysis at the European Centre for Medium Range Weather Forecasts (Cray)*

<https://www.seattletimes.com/business/technology/crays-supercomputers-advance-weather-forecasts/>

# Introduction

Chemical Transport Model (CTM)

Compute numerical model which simulates the atmospheric chemistry and transport

$$\frac{\partial C}{\partial t} = -\nabla \cdot (\mathbf{u} \cdot \mathbf{C}) + \frac{\partial}{\partial y} \left( K_v \frac{\partial C}{\partial y} \right) + E + R + Q - D - W$$

The diagram shows the equation for the Chemical Transport Model (CTM) with arrows pointing from descriptive labels to each term in the equation:

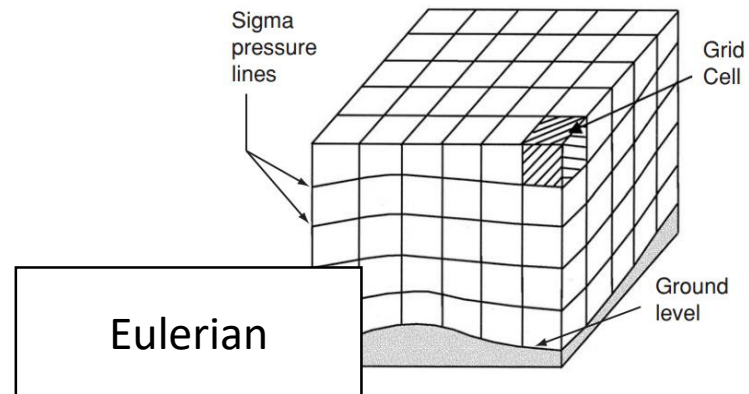
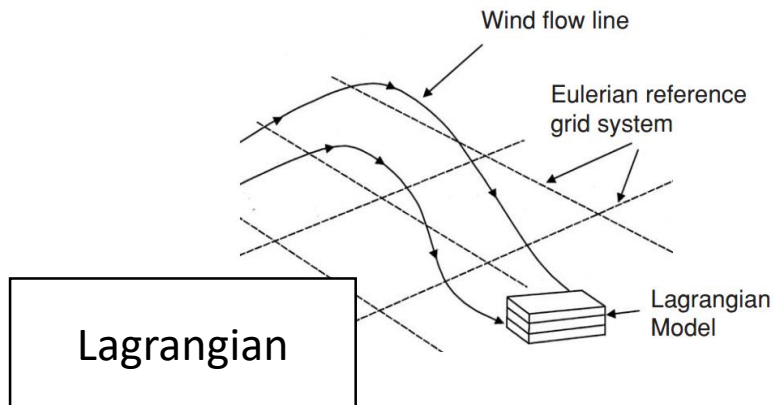
- $\frac{\partial C}{\partial t}$ : Change in concentration with time
- $-\nabla \cdot (\mathbf{u} \cdot \mathbf{C})$ : Grid resolved transport (Advection)
- $\frac{\partial}{\partial y} \left( K_v \frac{\partial C}{\partial y} \right)$ : Diffusion process
- $E$ : Entrainment and detrainment
- $R$ : Generation/Consumption chemical reactions
- $Q$ : Emissions
- $D - W$ : Dry and wet deposition processes

# Introduction

Chemical Transport Model (CTM)

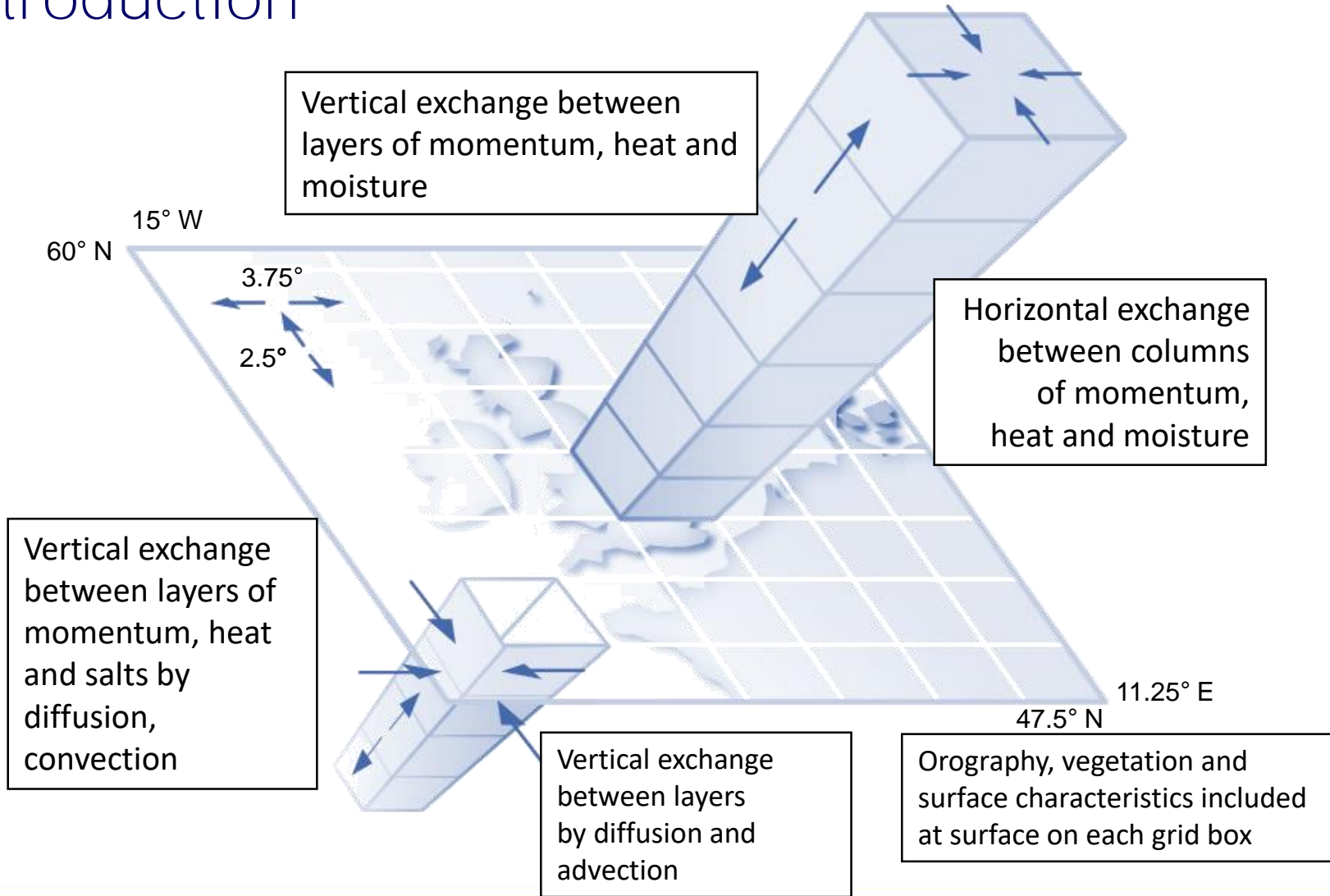
Compute numerical model which simulates the atmospheric chemistry and transport

$$\frac{\partial C}{\partial t} = -\nabla \cdot (\mathbf{u} \cdot \mathbf{C}) + \frac{\partial}{\partial \mathbf{v}} (K_v \frac{\partial C}{\partial \mathbf{v}}) + E + R + Q - D - W$$





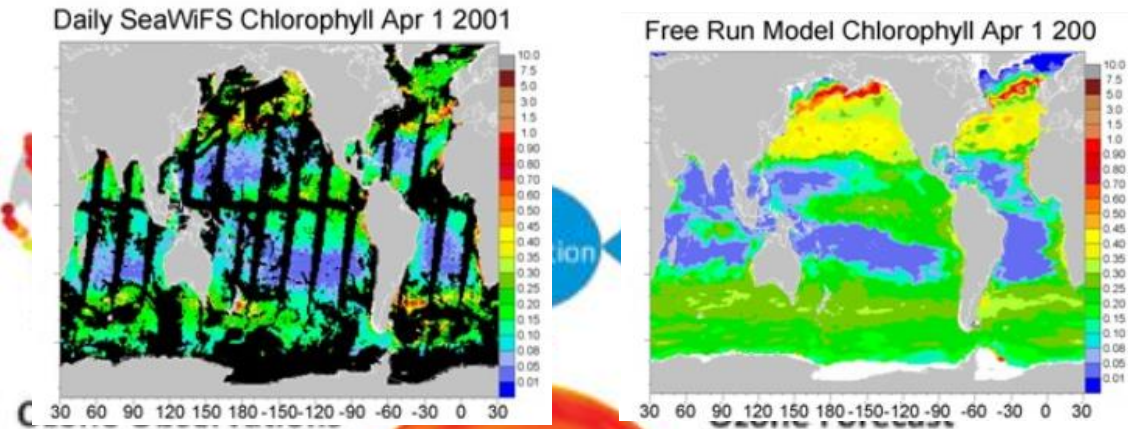
# Introduction



# Introduction

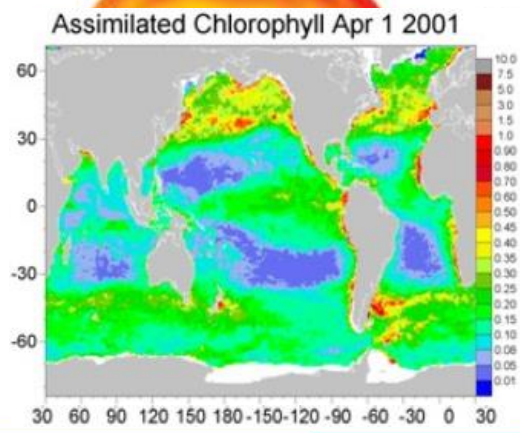
Data Assimilation is the process of absorbing (incorporate) observed information to improve a dynamical prognostic numerical model results

Information from Observations



Information from the model

+ errors      errors

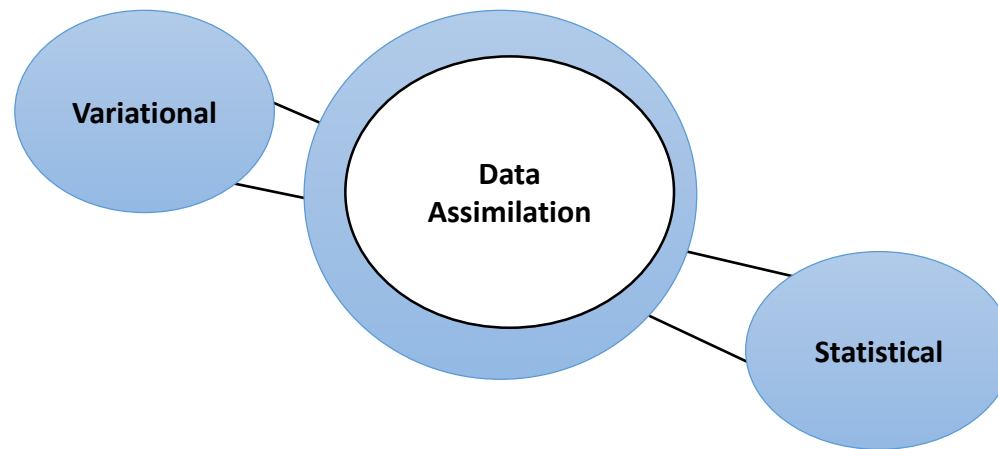


Combination of information through Data Assimilation

Improved analysis and forecast. example: Water temperatura, Ice coverage, wind velocity...

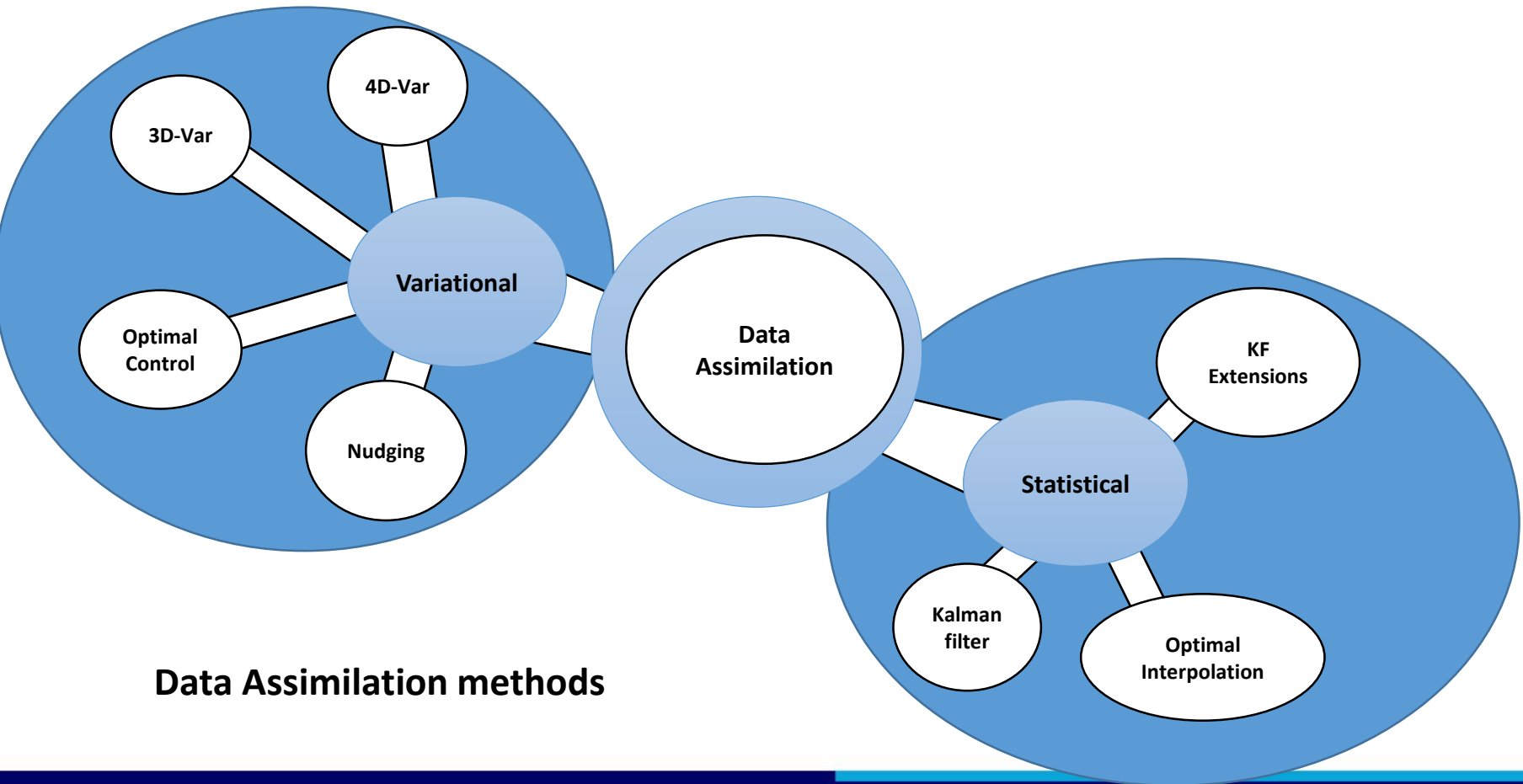
Gregg, W. (2007)

# Data Assimilation fundamentals

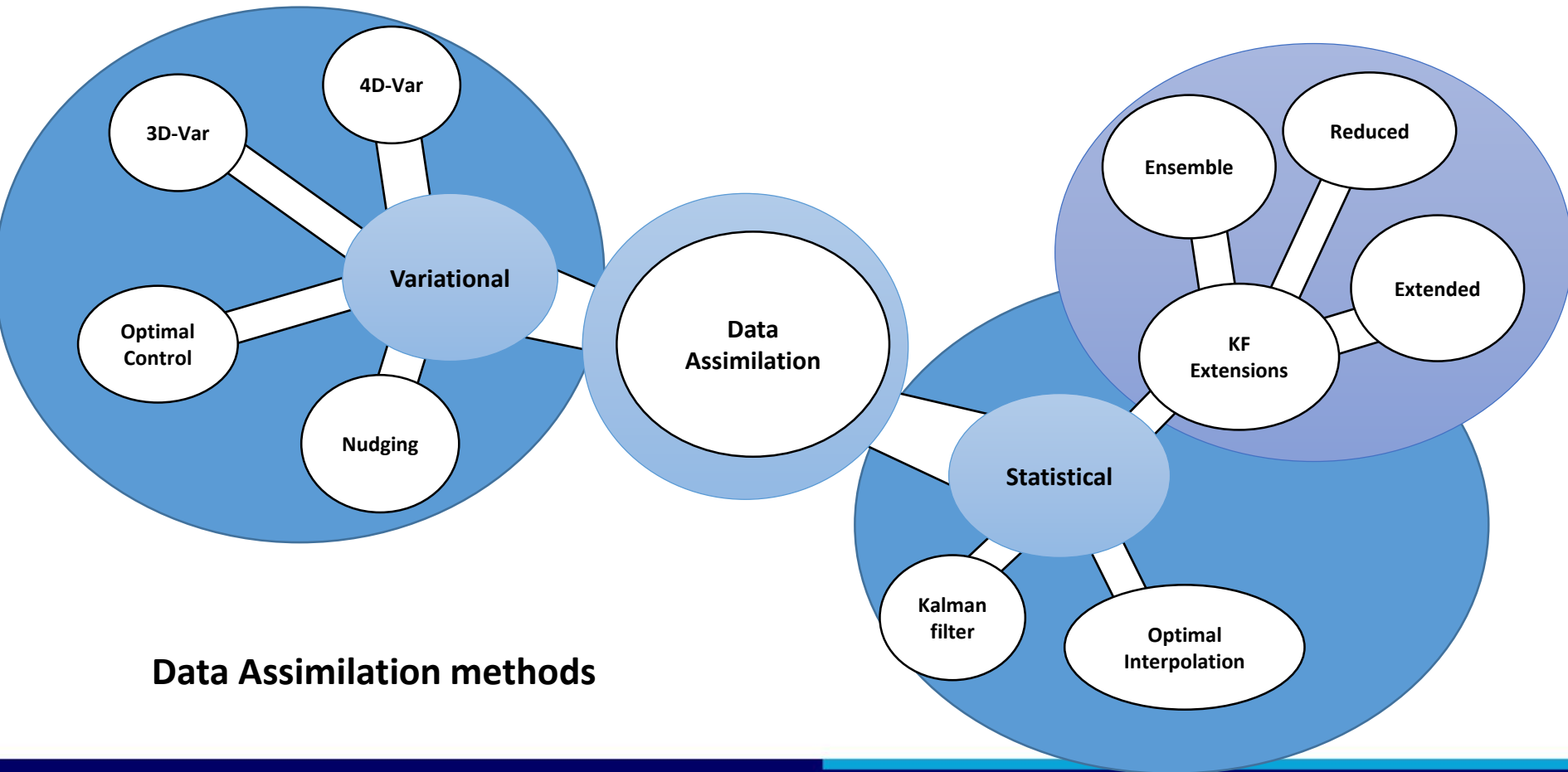


## Data Assimilation methods

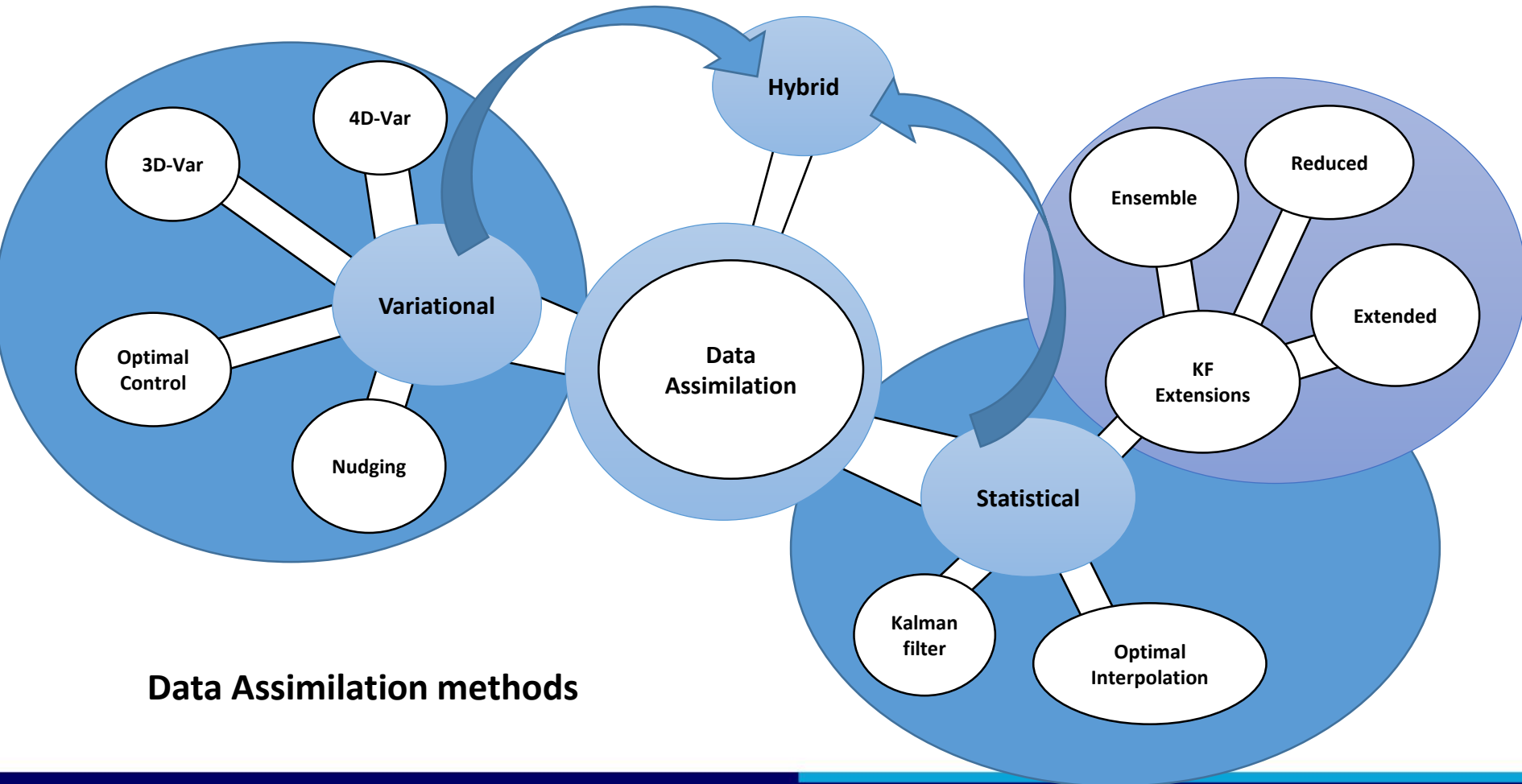
# Data Assimilation fundamentals



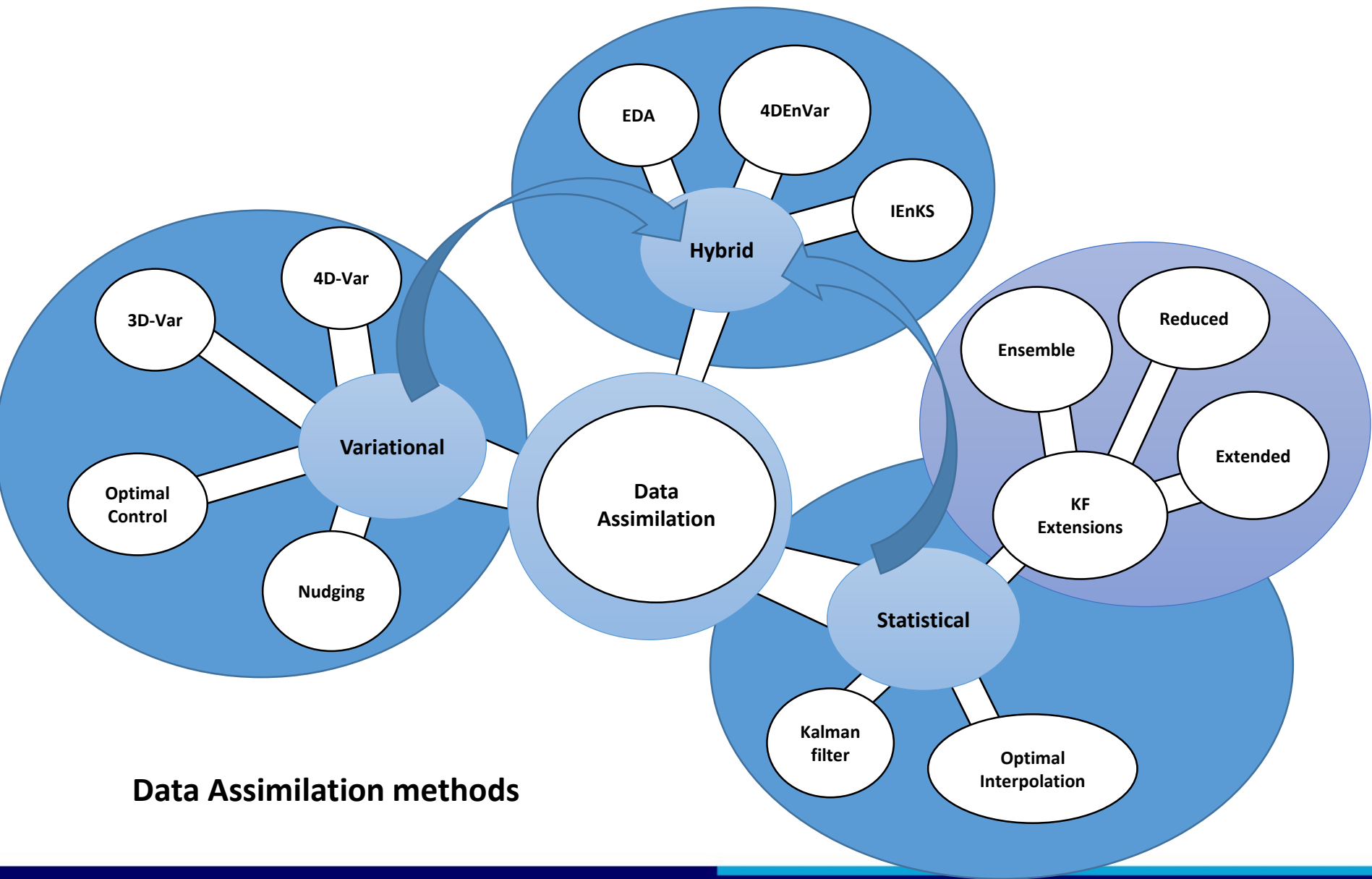
# Data Assimilation fundamentals



# Data Assimilation fundamentals



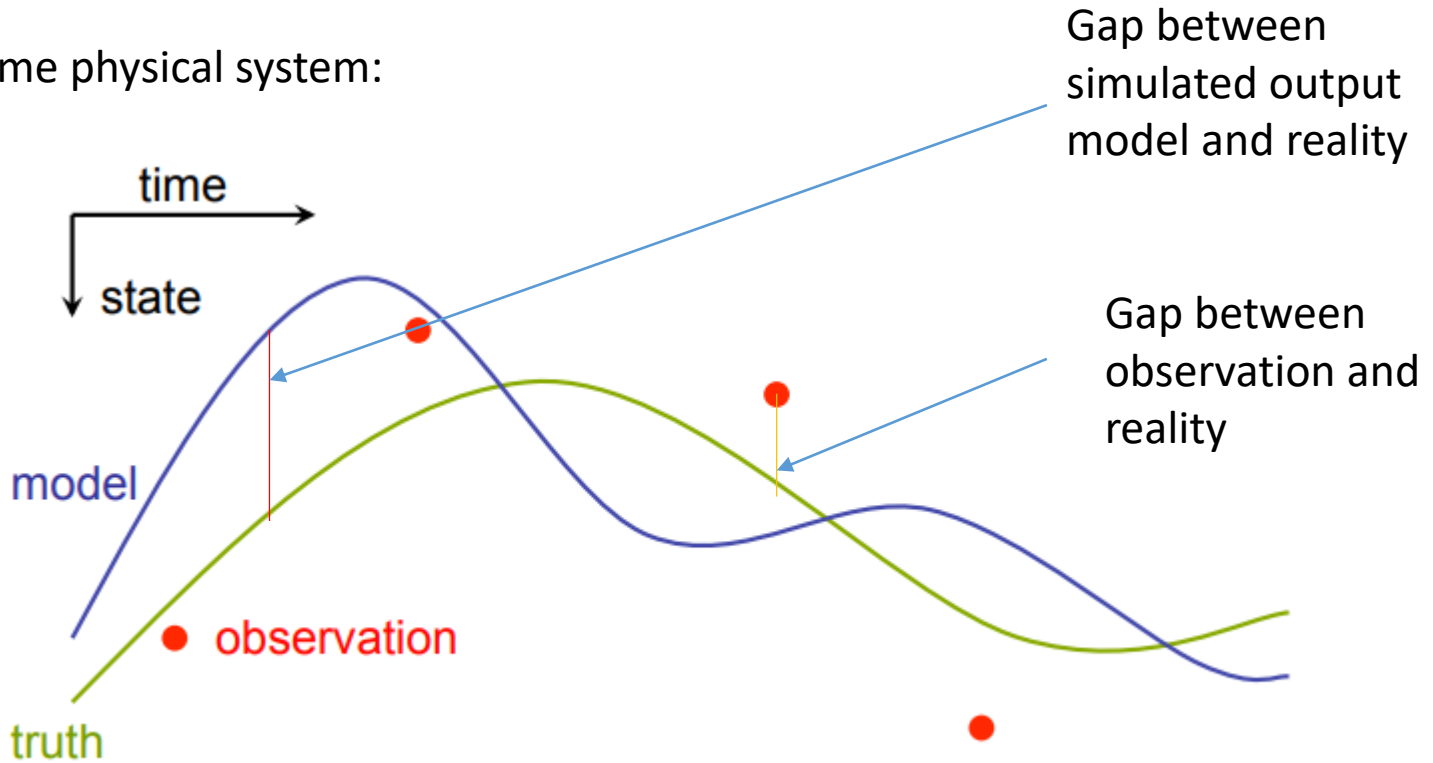
Data Assimilation methods



**Data Assimilation methods**

# Data Assimilation approaches

Consider some physical system:

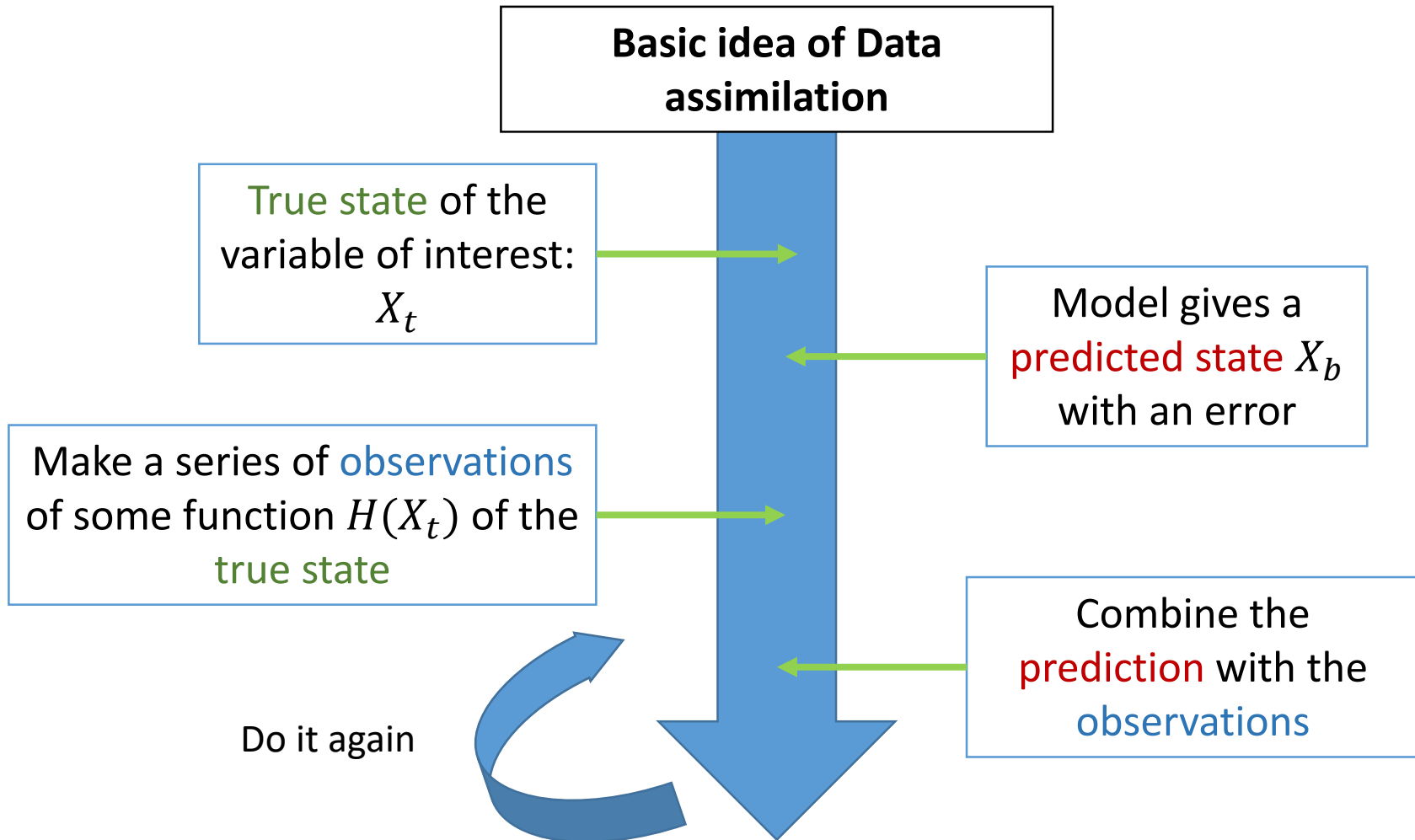


Sequential Assimilation

Variational Assimilation



# Data Assimilation approaches



# Variational approach

## Data Assimilation techniques

### Filter Techniques

Statistical optimization

**Kalman filter approach:** Sequential method that search to improve the model predictions **reducing the covariance error** between observations and model outputs

### Variational methods

Least squares error minimization

**Adjoint of the forward model:** Looks for the set of optimal states that **minimize cost functions** between observations made and model outputs

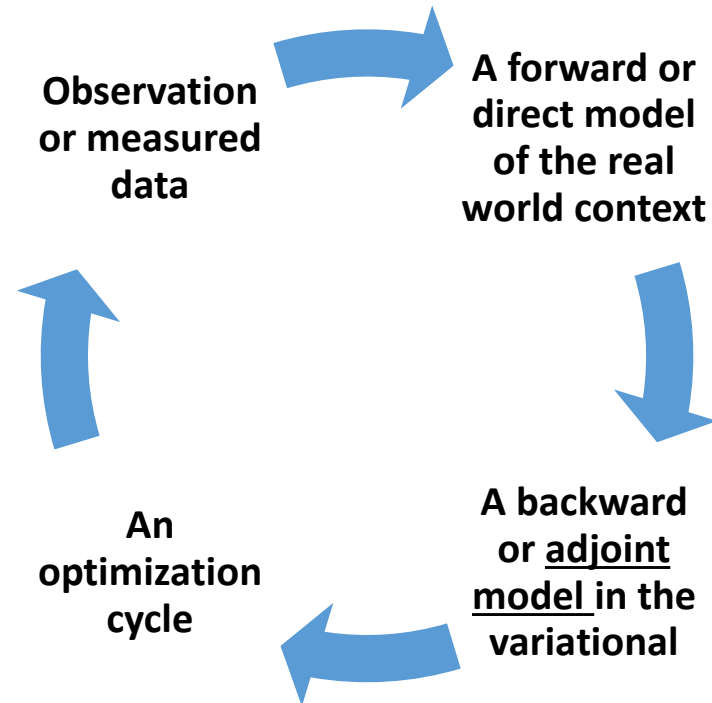
# Data Assimilation approaches

Inverse modelling:

The inverse modelling problem consists of using the actual result of some measurements **to infer** the values of the parameters that characterize the system. A. Tarantola (2005)

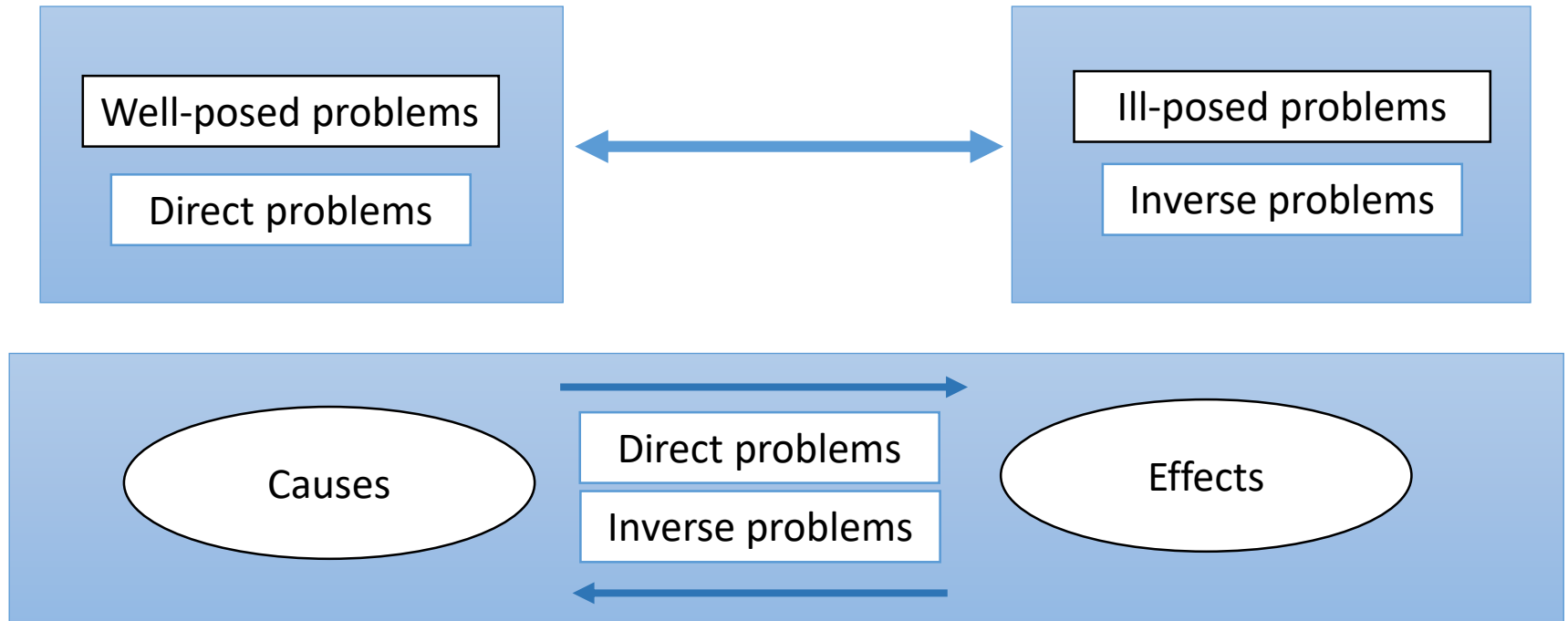
Four basic ingredients in any inverse or DA problem:

“Two problems are inverse to each other if the formulation of each involves all or part of the solution of the other” J.B.Keller (1966)



# Variational approach

Variational assimilation is based on **optimal control theory**



**Direct problem:** the computation of the trajectories of bodies from the knowledge of the forces.

**Inverse problem:** determination of the forces from the knowledge of the trajectories

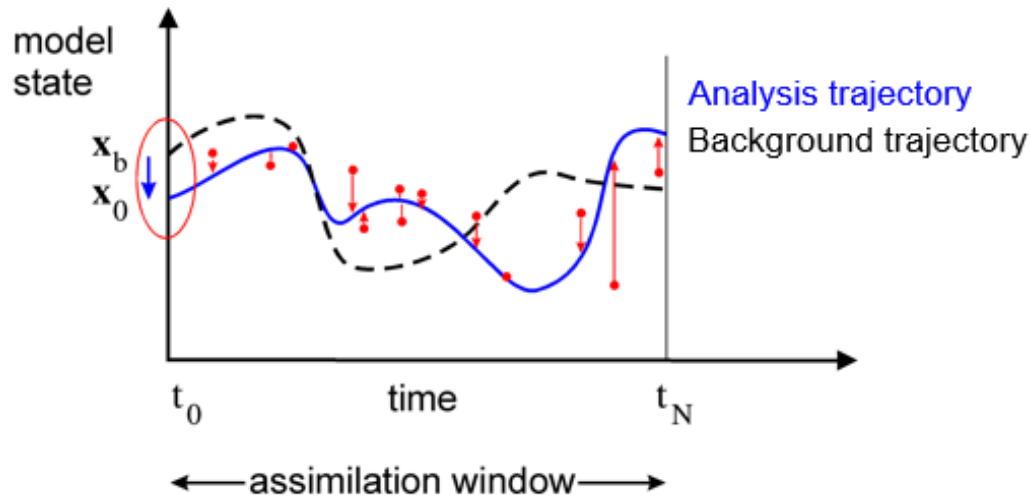
# Variational approach

Theory of **maximization** and **minimization**.

**Weiertrass theorem**: Every continuous function in a bounded domain attains a *maximal* and a *minimal* value inside the domain or on its boundary.

# Variational Data Assimilation

$$J(\mathbf{x}_0) = \frac{1}{2}(\mathbf{x}_0 - \mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_b) + \frac{1}{2}(H(\mathbf{x}_0) - \mathbf{y})^T \mathbf{R}^{-1}(H(\mathbf{x}_0) - \mathbf{y})$$



The **calculus of variations** deals with the following problema:

Find the *maximum* or *minimum* of a functional, over the given domain of admissible functions, for which the functional attains the **extremum** with respect to all argument functions in a small neighborhood of the extremal argument function.

# Variational approach

Cost function  $J(\mathbf{m})$  as an **energy functional**

**3D-Var**

$$J = \min \frac{1}{2} [(\mathbf{x}_0 - \mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x}_0 - \mathbf{x}_b) + (H(\mathbf{x}_0) - \mathbf{y})^T \mathbf{R}^{-1} (H(\mathbf{x}_0) - \mathbf{y})]$$

Distance to forecast

Distance to observations

**4D-Var**

$$J = \min \frac{1}{2} [(\mathbf{x}_0 - \mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x}_0 - \mathbf{x}_b) + \sum_i^s (H(\mathbf{x}_i) - \mathbf{y}_i)^T \mathbf{R}^{-1} (H(\mathbf{x}_i) - \mathbf{y}_i)]$$

Distance to forecast

Distance to observations

A misfit **functional** that quantifies the distance between the observation and the model prediction.

# Variational approach

Which approach, Kalman filtering or variational assimilation, is better?



# Previous Results



Prof.dr.ir. A.W.  
Heemink



**Fu, G. (2016).** Improving volcanic ash forecast with ensembled-based data assimilation (doctoral dissertation). Technische Universiteit Delft, Delft, The Netherlands.

**Lu, S. (2016).** *Variational data assimilation of satellite observations to estimate volcanic ash emission* (doctoral dissertation). Technische Universiteit Delft, Delft, The Netherlands.

# Previous Results

TuDelft PhD Theses

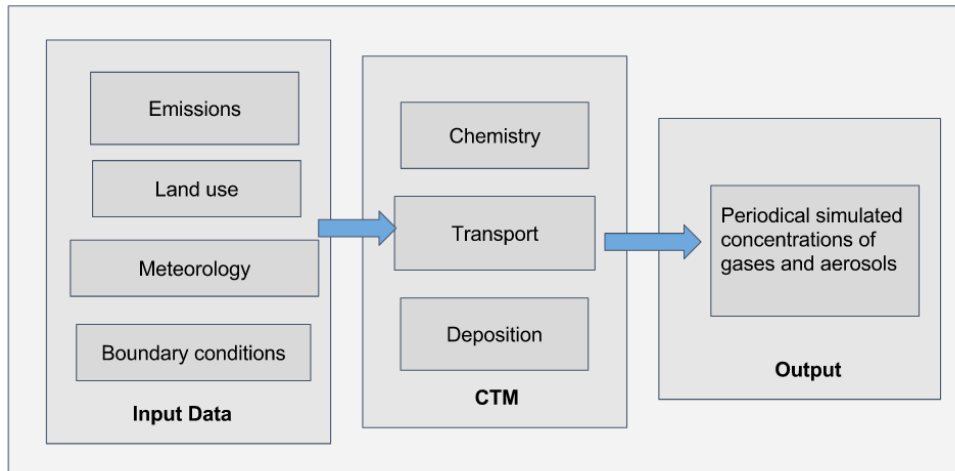


Eyjafjallajokull volcano  
eruption plume

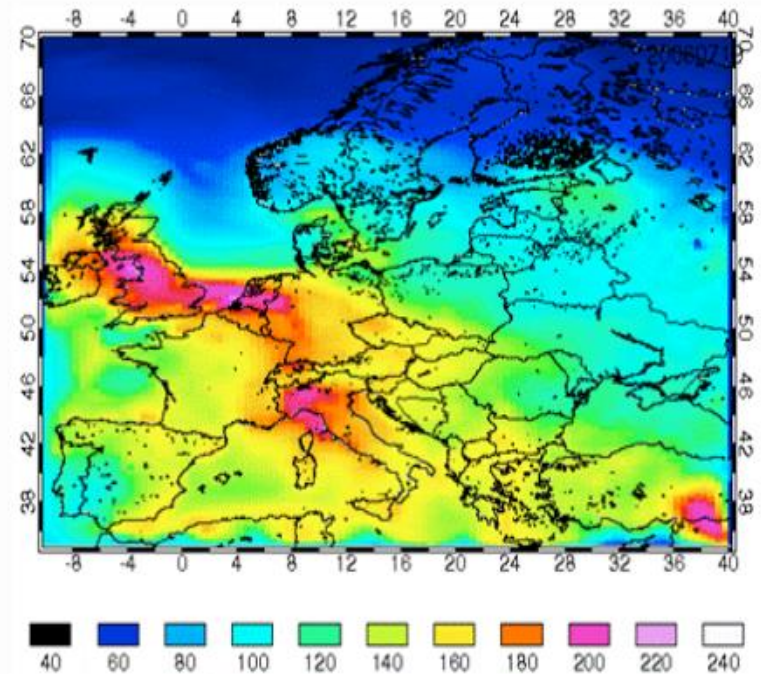
The **April-May 2010** eruption of Eyjafjallajokull volcano (Iceland) caused an unprecedented closure of the European and North Atlantic airspace with global economic losses of **5 billion US dollars**

Oxford-Economics (2010)

# Previous Results



Chemical Transport Model (CTM)



LOTOS EUROS Output for NO<sub>2</sub>

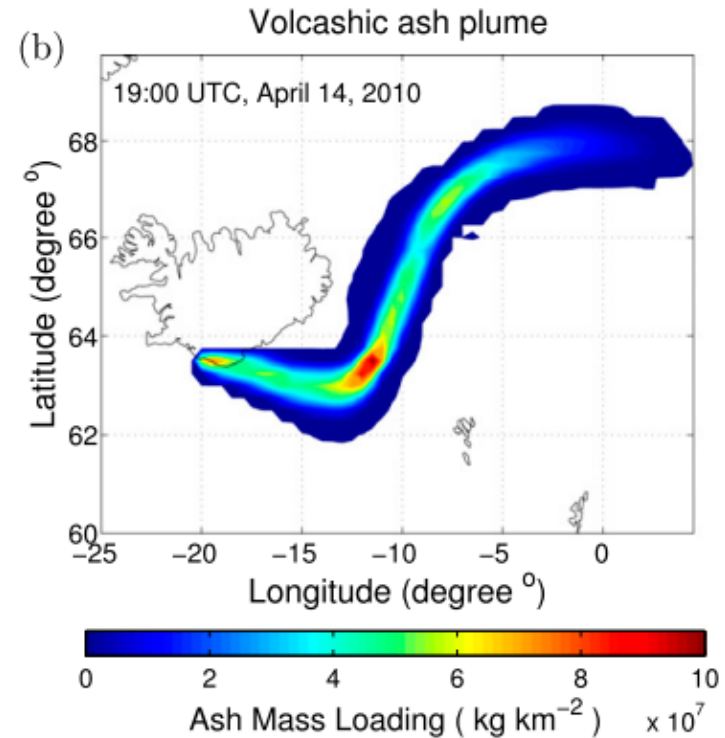
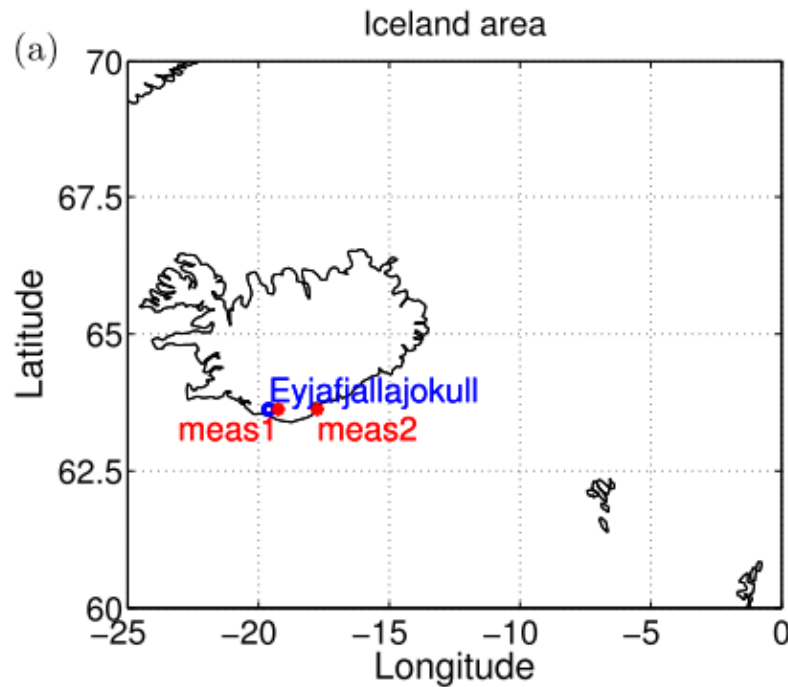
# Previous Results



Image extracted from <https://www.youtube.com/watch?v=bjAOqMf3DUY&t=119s>

# Previous Results

TU Delft Doctoral dissertations

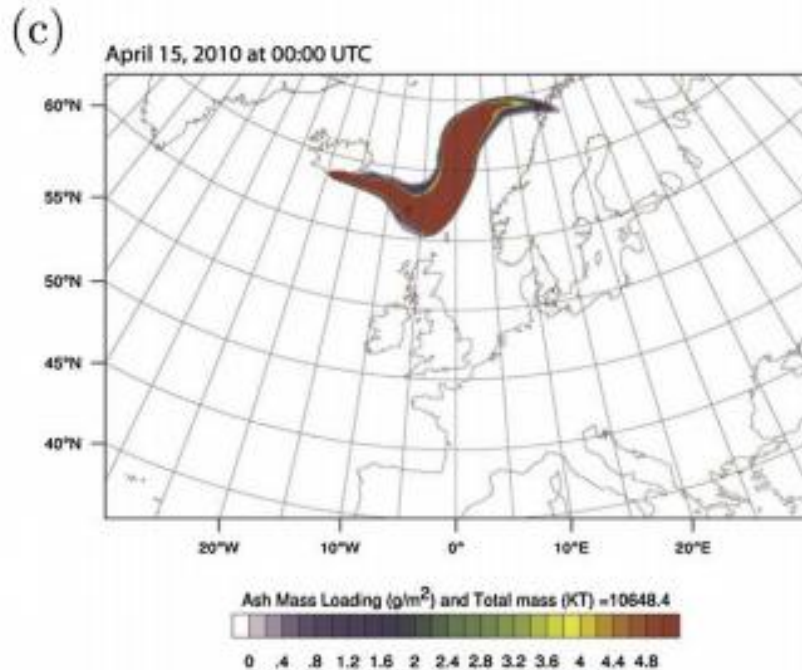


(Lu. S. (2016). Fu. S (2016).)

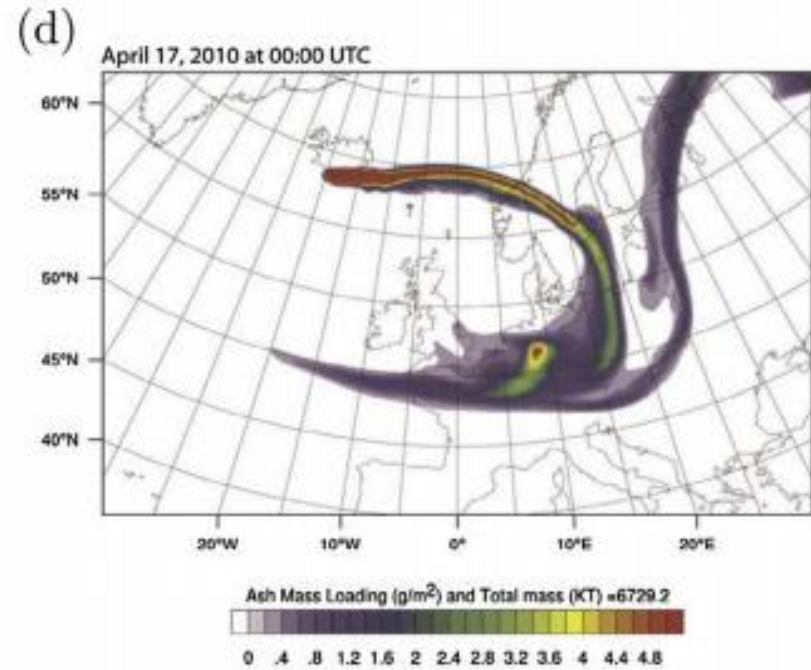
# Previous Results

TU Delft Doctoral dissertations

April 14-18, 2010



April 15, 2010



April 17, 2010

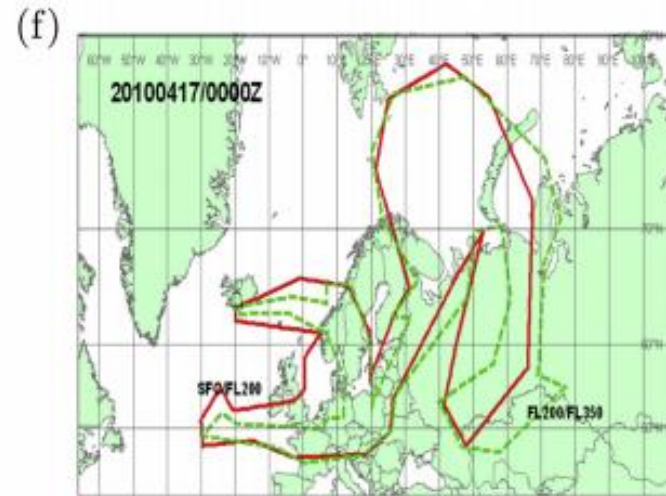
~~MOROSHEIROS~~

# Previous Results

TU Delft Doctoral dissertations



April 15, 2010



April 17, 2010

Volcanic Ash Advisory Center (VAAC) based on the **NAME** model

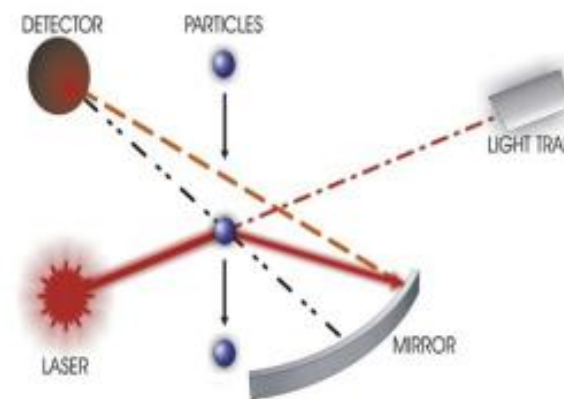
# Previous Results

TU Delft Doctoral dissertations

(a)



(b)



OPC Optical Particle Counter



# Previous Results

TU Delft Doctoral dissertations

Variations of the proposed DA methods

**Lu, S. (2016).**

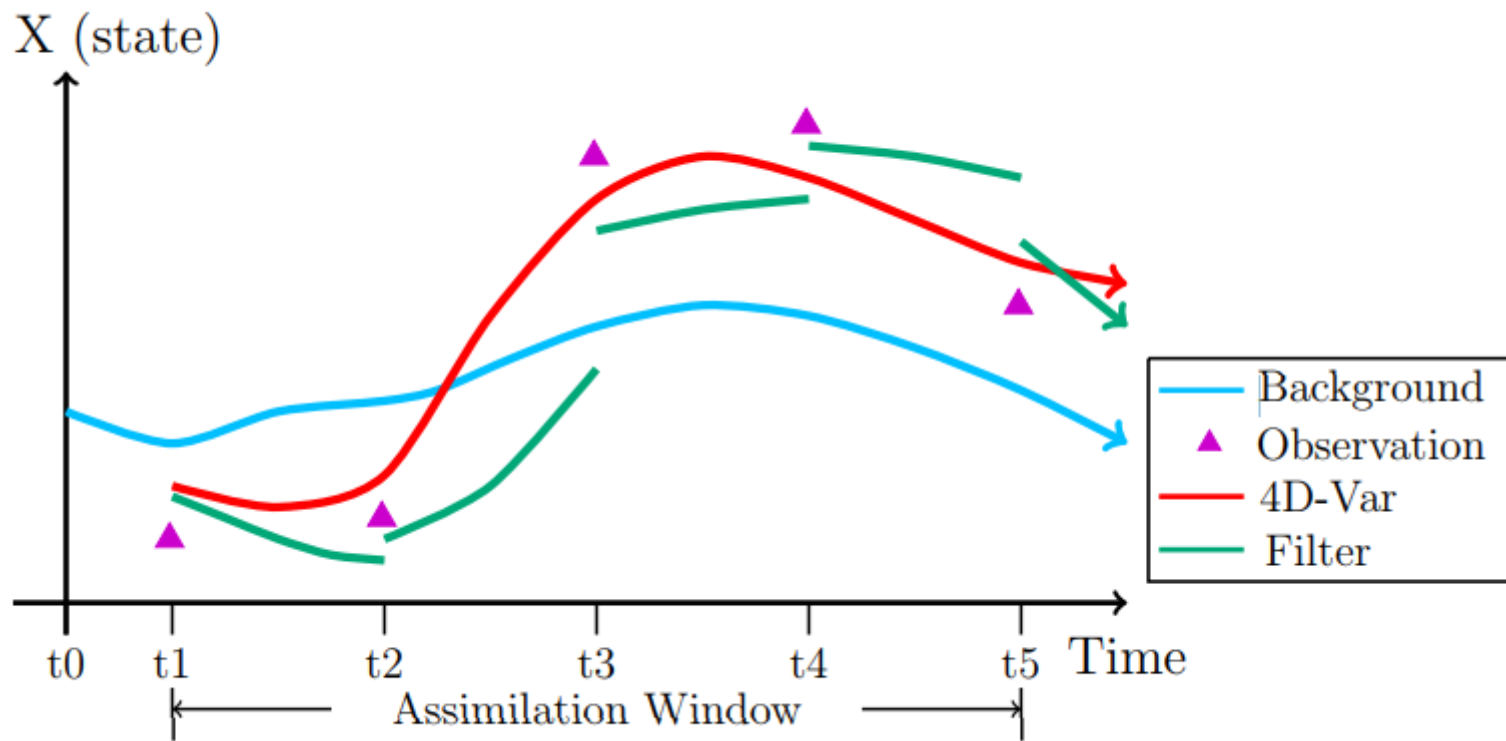
Trajectory 4D-Var

**Fu, G. (2016).**

Two-way-tracking  
Localized EnKF (TL-EnKF)

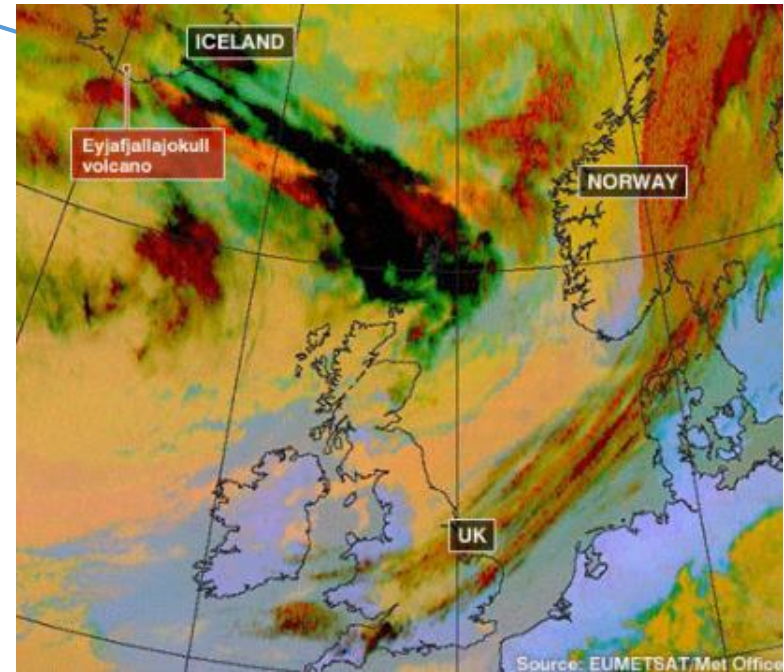
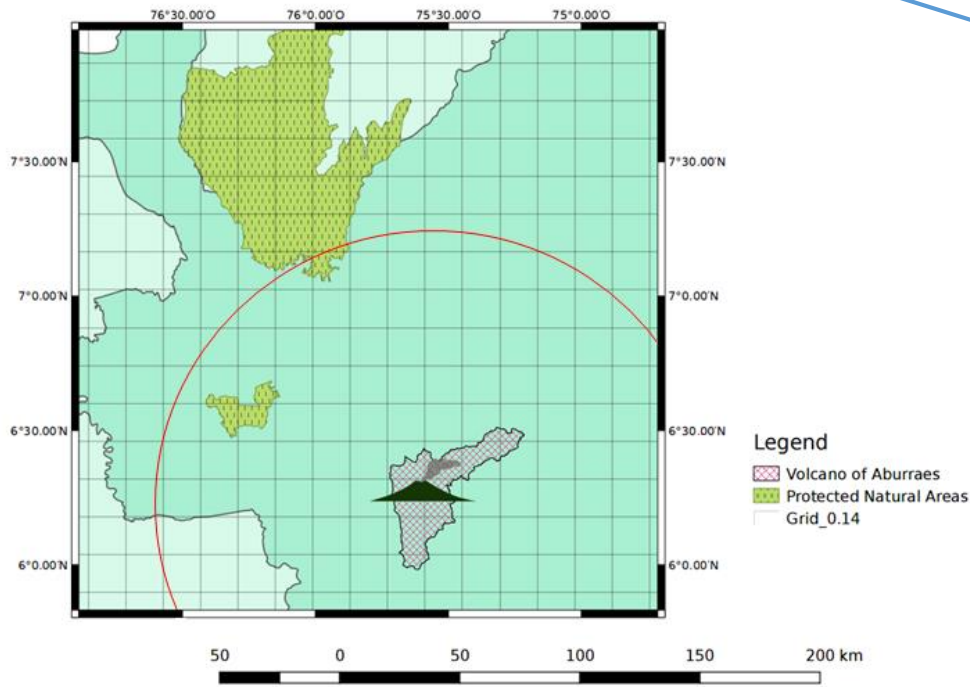
# Previous Results

TuDelft PhD Theses



# Current and future work

“The Volcano of the Aburraes”



teleconnection

Analogy of the **volcano of the Aburraes** with the volcano of Iceland

# Current research questions

How to generalize and formalize, through the help of Control Systems formalism, the answers to the Data Assimilation questions regarding the **volcano of Aburraes**.

How can we try or propose a **hybrid scheme** for the ill-conditioned problem in order to deal with reduced **observational noise**.

How to develop further evidence and formalize the postulate of accuracy of the method for a physical-related window of assimilation, regarding the demonstrated accuracy in assimilation windows that does not compromise the constant concentration of particles in atmosphere (Lu et al, 2016 under review).

# Current research questions

Is it possible to use a formal sensitivity analysis to analyze the perturbation of the inputs to the two experience based modifications in order to the penalty term to develop a generalized method and probe the stability of the solution for the **trj-4DVar**?

Verify the feasibility for the development of an extension for the **trj-4DVar** in LOTOS-EUROS Volcanic ash problem to bigger systems and mathematically feasible solutions to solve the modified cost function in a new or improved approach **adjoint free** for the case of the Aburrá Valley

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Fu. S. et al. (2016). Improving volcanic ash forecast with ensemble-based data assimilation. TuDelft Phd thesis.

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Páll Bergthörsson & Bo R. DÖÖs (1955) Numerical Weather Map Analysis, Tellus, 7:3, 329-340, DOI: 10.3402/tellusa.v7i3.8902



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Seubers H. et al. (2013) Data assimilation for OpenFOAM, Combining measurements and modelling. Aerospace Engineering. TuDelft. Presentation Deltares

Tarantola A. et al. (2005). Inverse problema theory and methods for model parameter estimation. SIAM Society for Industrial and Applied Mathematics. ISBN 0-89871-572-5.