Optimization approximation with separable variables for the one-way wave operator

Héctor Román Quiceno E.
Asesor
Jairo Alberto Villegas

Doctorado en Ingeniería Matemática
Universidad EAFIT
C3-Instituto Tecnológico Metropolitano

May 2016

Outline

Outline

Outline

Outline

The thin slab propagator

- The thin slab propagator
- Reduced system
- The coupled system of one-way equations
- The propagator

The thin slab propagator

- The thin slab propagator
- Reduced system
- The coupled system of one-way equations
- The propagator

The thin slab propagator

- The thin slab propagator

$$\partial_k \hat{p} + s\rho \hat{v}_k = \hat{f}_k$$
$$s\kappa \hat{p} + \partial_r \hat{v}_r = \hat{q}$$

where

$$\hat{p}(x_m, s) = \mathcal{L}[p(x_m, t)]$$
$$\kappa \quad \text{compressibility modulus} \tag{1}$$

- Reduced system
- The coupled system of one-way equations
- The propagator

The thin slab propagator

- The thin slab propagator
- Reduced system
- The coupled system of one-way equations
- The propagator

The thin slab propagator

- The thin slab propagator
- Reduced system
  to handle horizontal velocity of the particle

$$\hat{v}_\mu = -\rho^{-1} s^{-1} (\partial_\mu \hat{p} - \hat{f}_\mu)$$

- The coupled system of one-way equations
- The propagator

The thin slab propagator

- The thin slab propagator
- Reduced system
  from which we obtain the matrix differential equation

$$(\partial_3 \delta_{I,J} + s\hat{A}_{I,J})\hat{F}_J = \hat{N}_I$$

$$\begin{aligned} \hat{F}_1 = \hat{p} \quad , \quad \hat{F}_2 = \hat{v}_3 \\ \hat{A}_{1,1} = \hat{A}_{2,2} = 0 \\ \hat{A}_{1,2} = \rho \\ \hat{A}_{2,1} = -D_\nu(\rho^{-1} D_\nu) + \kappa \\ \hat{N}_1 = \hat{f}_3 \quad , \quad \hat{N}_2 = D_\nu(\rho^{-1} \hat{f}_\nu) + \hat{q} \end{aligned}$$

- The coupled system of one-way equations
- The propagator

The thin slab propagator

- The thin slab propagator
- Reduced system
- The coupled system of one-way equations
- The propagator

The thin slab propagator

- The thin slab propagator
- Reduced system
- The coupled system of one-way equations

$$(\partial_3 \delta_{I,M} + s\hat{\Lambda}_{I,M})\hat{W}_M = -(\hat{L}^{-1})_{I,M}(\partial_3 \hat{L}_{M,K})\hat{W}_K + (\hat{L}^{-1})_{I,M}\hat{N}_M$$

- The propagator

The thin slab propagator

- The thin slab propagator
- Reduced system
- The coupled system of one-way equations
- The propagator

The thin slab propagator

- The thin slab propagator
- Reduced system
- The coupled system of one-way equations
- The propagator
  For the coupled system, we can get the operator

$$(\hat{G}\hat{u})(x_\mu, x_3) = \int_{\zeta\in\mathcal{R}} \int_{x`_\nu\in\mathcal{R}} \hat{\Upsilon}(x_\mu, x_3; x_\nu`, \zeta)\hat{u}(x_\mu`, \zeta) dx_1`dx_2`d\zeta$$

  with the initial conditions

$$(\partial_3 + s\hat{\Gamma})\hat{U} = 0 \quad , \quad \text{for} \quad x_3 > \zeta \quad , \quad \hat{U}(x_{\mu,\zeta;\zeta}) = \hat{u}(x_\mu, \zeta)$$

  we get

$$(\hat{G}\hat{u})(x_\mu, x_3) = \int_{\zeta=-\infty}^{x_3} \hat{U}(x_\mu, x_3; \zeta) d\zeta$$

  where

$$\hat{U}(x_\mu, x_3; x_3`) = \pm H(\mp[x_3` - x_3]) \left\{ \prod_{\zeta=x_3`}^{x_3} exp[-s\hat{\Gamma}(x_\mu, \zeta) d\zeta] \right\} \hat{u}(x_\mu, x_3`)$$

The thin slab propagator

- The thin slab propagator
- Reduced system
- The coupled system of one-way equations
- The propagator
  then ,for a thin slab ($\triangle x_3$, sufficiently small), we have the propagator

$$\hat{p}(\alpha_\mu, x_3; \alpha_\nu``, x_3`) \simeq \int exp[is(\alpha_\sigma - \alpha_\sigma``)x_\alpha]exp[-s\hat{\gamma}(x_\mu, x_3 - \frac{1}{2}\triangle x_3, \alpha_\nu``, s)\triangle x_3]dx_1 dx_2$$

The approximation method

The approximation method

Consider the 3-D acoustic one-way equation

$$\frac{\partial}{\partial z}U(x,y,z,\omega) = i\sqrt{\frac{\omega^2}{c^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}}\,U(x,y,z,\omega)$$

The approximation method

for which, the thin slab propagator is

$$g(x, y, z; x', y'z') \simeq \frac{1}{4\pi^2} \int exp\left[i\sqrt{\frac{\omega^2}{c^2(x, y, z'')} - (k_x^2 + k_y^2)} \triangle z\right] \cdot exp[i(k_x(x-x')+k_y(y-y'))$$

where $z'' = z' + \frac{1}{2} \triangle z$

The approximation method

Let us take the kernel

$$\mathcal{A}(u,k) = exp\left[i\sqrt{u^2 - k^2}dz\right]$$

$$u = \frac{\omega}{c(x,y,z)} \quad , \quad k^2 = k_x^2 + k_y^2$$

The approximation method

- the optimization approximation method is
- we get

The approximation method

- the optimization approximation method is
  To find functions $\phi(u)$, $\psi(k)$ and a complex number λ, such that

$$||\mathcal{A}(u,k) - \lambda\phi(u)\psi(k)^*||_{L^2} = min_{\bar{\phi},\bar{\psi},\bar{\lambda}}||\mathcal{A}(u,k) - \bar{\lambda}\bar{\phi}(u)\bar{\psi}(k)^*||_{L^2}$$

  where

$$\bar{\phi} \in \{\bar{\phi}(u) \in L^2[a,b], ||\phi(\bar{u})|| = 1$$
$$\bar{\psi} \in \{\bar{\psi}(u) \in L^2[c,d], ||\psi(\bar{u})|| = 1$$

$$\int_c^d \mathcal{A}(u,k)\psi(k)dK = \lambda\phi(u)$$
$$\int_a^b \mathcal{A}(u,k)^*\phi(u)du = \lambda^*\psi(k)$$

The approximation method

- the optimization approximation method is

  Which is transformed into an independent self-adjoint system of integral equations, given by

$$\int_a^b \int_c^d \mathcal{A}(u,k)\mathcal{A}(\bar{u},k)^* \phi(\bar{u}) d\bar{u} dk = |\lambda|^2 \phi(u)$$
$$\int_a^b \int_c^d \mathcal{A}(u,k)^* \mathcal{A}(u,\bar{k}) \psi(\bar{k}) du d\bar{k} = |\lambda|^2 \psi(k)$$

The approximation method

- the optimization approximation method is

  this integrals can be approximated by

$$\sum_{i=1}^{m}\left[\sum_{j=1}^{n}\triangle u \triangle k \mathcal{A}(u_l,k_j)\mathcal{A}(u_i,k_j)^*\right]\phi_i$$

The approximation method

- the optimization approximation method is

  then, we have

$$F\phi = |\lambda|^2\phi$$

$$G\psi = |\lambda|^2\psi \quad \text{for the second equation.}$$

The approximation method

- the optimization approximation method is

  Clearly

$$F = \mathcal{A}\mathcal{A}^H$$
$$G = \mathcal{A}^H\mathcal{A}$$

The approximation method

- the optimization approximation method is

  and we have the approximation

$$\mathcal{A}(u,k) \simeq \lambda_1 \phi_1(u) \psi_1(k)^*$$

The approximation method

- the optimization approximation method is

  which can be optimized as

$$\mathcal{A}(u,k) \simeq \sum_{l=1}^{s} \lambda_l \phi_l(u) \psi_l(k)^*$$

The approximation method

- the optimization approximation method is
- then the propagator is

The approximation method

- the optimization approximation method is
- then the propagator is

$$g(x, y, z; x', y'z') = \frac{1}{4\pi^2} \sum_{l=1}^{s} \lambda_l \phi_l(u) \int \psi_l(k)^* exp[i(k_x(x-x')+k_y(y-y'))]dk_x dk_y$$

Some references

1. M De Hoop; J Le Rousseau. 1999.*Generalization of the phase-screen approximation for the scattering of acoustic waves*. CWP,1-44.
2. Jing-Bo; Hong Liu. 2004. *Optimization approximation with separable variables for the one-way operator*: Geophysical Research Letters, 31.
3. J. Song.2001.*The optimization expression of functions and manifolds in high dimensions by ones in low dimensions*. Chin. Sci. Bull, 46, 977-984.